maintain correct gap heights during high temperature operation, the peakiness of the outlet velocity profile would tend to be further increased. If the flow pattern postulated for splash devices is correct, then the development of flow near the outlet from such slots should be less sensitive to velocity ratio than that from total-head devices. Measurements of potential core have shown this tendency, although, in fairness, this also could be because of other factors. The existence of initial velocity profiles in the film would result in extended transition regions, as indicated by Eq. 3.

For this particular application, it can thus be concluded that treatment of the cooling film as a turbulent boundary layer is unlikely to be rewarding, and development of different mass entrainment laws will be required for Spalding's new theory.[6] Investigations into these aspects are being carried out at this establishment as part of a broader program of combustion chamber cooling research. To paraphrase Spalding, obviously for dirty geometry slots, a flow that is of the boundary layer type away from the slot can be considered jet-like near the slot outlet, for the range of velocity ratios likely to be encountered. Application of Spalding's formula should be limited to the simple geometry slots for which it is valid and was intended, where initial velocity profiles in the film are vanishingly small and there is no wake from the lip of the slot.

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Comments on "Stresses about a Circular Hole in a Cylindrical Shell"

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It seems to be worthwhile to add some remarks on the behavior of stresses around the hole for large values of the curvature parameter β.

The author states[1] that the Hankel functions of the first kind behave like

$$H_n{}^1 \sim Ce^{-\beta r}/(\beta r)^{1/2} \tag{33}$$

for large arguments, so that the stress function behaves like

$$\phi^* \sim Ce^{-\beta r(1 - |\cos\theta|)}/(\beta r)^{1/2} \tag{34}$$

Received December 15, 1965.

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This implies that the stresses caused by the hole at the 90° position decay most rapidly.

On the other hand, the boundary-layer analysis results in the following behavior of the stress function ϕ^* for large β:

$$\phi^* \sim Ce^{-2(1-i)\beta r\cos\theta} \qquad \text{for} \qquad \cos\theta > 0 \tag{52}$$

thus implying that stresses caused by the hole at the 0° position decay most rapidly, whereas, on the contrary at $\theta = 90°$, a turning-point-type singularity appears. This conclusion seems to be the correct one. It was to be expected on the basis of the general asymptotic theory of shells and the singularity was pointed out, for instance, in Ref. 2.

The contradiction between (34) and (52) arises because the asymptotic formula (33) is valid for large values of the argument provided the order n of the Hankel function is small.†[3]

However, since "as β was increased, the number of terms needed in the series to obtain converging results also increased,"[1] and, for $\beta = 4$ for instance $n \leqq 28$, the asymptotic formula (33) remains valid only at very large distances from the hole, where $r \gg 1$, and not near the hole, where $\beta r \approx \beta$.

References

[1] VanDyke, P., "Stresses about a circular hole in a cylindrical shell," AIAA J. **3**, 1733–1742 (1965).

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† This was pointed out to the author of this note by D. A. Ludwig of the Courant Institute.

Reply by Author to A. Kornecki

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The author is indebted to A. Kornecki for mentioning a point that was omitted in the original paper: the behavior of the stresses are characterized by the behavior of Eq. (34)[1] in the region near the hole when the parameter β is small enough so that the order n of the last term in the required Hankel function series also is small. This behavior, where the stresses at the 90° position decay most rapidly, is still evident at a β of 2(an n of 9) as indicated by Figs. 6 and 11; the stresses have reached the values of the membrane solution most quickly at the 90° position as compared with the 45° and 0° positions.

Further illumination regarding the behavior of the stresses in the shell as β becomes large is clearly necessary. The stress behavior in possible side ($|\cos\theta| > 0$) boundary layers, as characterized by Eq. (52), is, however, of interest only in the case of pressure loading; this side boundary-layer behavior is absent in the cases of loading by tension and torsion. A complete evaluation of the decay of the stresses at the hole edge, with the resulting determination of the point at which this decay is most rapid, would still appear to the author to depend upon a solution to the differential equation which governs the boundary layer at the 90° positions on the hole

$$\Phi^*{}_{,\bar{y}\bar{y}\bar{y}\bar{y}} + 8i(\Phi^*{}_{,\bar{x}\bar{x}} + \bar{x}^2\Phi^*{}_{,\bar{y}\bar{y}} + 2\bar{x}\Phi^*{}_{,\bar{x}\bar{y}} + \Phi^*{}_{,\bar{y}}) = 0 \tag{58}$$

under the boundary conditions [Eqs. (59) and (67)] appropriate to the loading.

Reference

[1] VanDyke, P., "Stresses about a circular hole in a cylindrical shell," AIAA J. **3**, 1733–1742 (1965).

Received January 7, 1966.

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